

# Using Abstraction for Generalized Planning

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## Abstract

Given the complexity of planning, it is often beneficial to create plans that work for a wide class of problems. This facilitates reuse of existing plans for different instances of the same problem or even for other problems that are somehow similar. We present novel approaches for learning, and even finding such plans using state representation and abstraction techniques originally developed for static analysis of programs. The *generalized plans* that we compute include loops and work for a large class of problem scenarios having varying numbers of objects that must be manipulated to reach the goal. Our algorithm for learning generalized plans takes as its input an example plan for a certain problem instance and a finite 3-valued first-order structure representing a set of initial states from different problem instances. It learns a generalized plan along with a classification of the problem instances where it works. The algorithm for finding plans takes as input a similar 3-valued structure and a goal test. Its output is a set of generalized plans and conditions describing the problem instances for which they work.

## Introduction

In this paper we develop a new unified framework for finding and learning generalized plans. We use it to construct novel algorithms for finding or learning provably correct plans for sets of problem instances from a domain. These problem instances could not only differ in the number of elements which need to be manipulated for reaching the goal, but also have no bounds on these numbers. This is accomplished by including loops over such objects. Our plans are thus closer to algorithms: our input represents an abstract planning problem and the generalized plans we compute solve it for a range of problem instances. We also present a precise analytical characterization of the domains (*extended-LL* domains) where our techniques are guaranteed to work.

This paper is organized as follows. We first provide a high level overview of our technique, followed by sections describing the abstraction mechanism, our methodology and the requirements we impose in order to identify a useful category of domains that we currently handle. This is followed by sections on learning generalized plans from examples and on finding them from scratch. Since this paper is restricted to 6 pages, all proofs and most of the details can be found at [Srivastava *et al.*, 2007]. To save space, we provide only a sketch of the TVLA system. We suggest the reader use our discussion of TVLA as an overview for the description in [Sagiv *et al.*, 2002].

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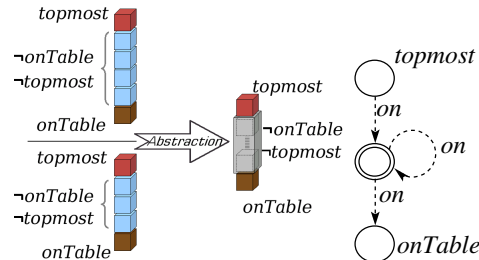


Figure 1: Canonical abstraction in blocks world. Abstracted objects are encapsulated. The abstraction predicates are *topmost* and *onTable*; diagram on the right shows the state in TVLA notation.

## Overview of the Approach

Our approach is based on *canonical abstraction* that has been used effectively in the Three-Valued Logic Analyzer (TVLA) – a tool for static analysis of programs that manipulate pointers [Sagiv *et al.*, 2002; Loginov *et al.*, 2006]. Canonical abstraction groups together any objects that are the same with respect to certain key properties: unary predicates referred to as *abstraction predicates*. The values of all the abstraction predicates on an object of the domain define the *role* that it plays. Abstract states are then generated by merging all objects in a role into a single summary element. For example, Fig.1 shows the abstraction predicates, roles, and an abstract representation for a blocks world domain.

We need a sound methodology for modeling actions with such abstract representations. This is done in TVLA using action transformers specified in first-order logic with transitive closure, discussed in detail in the next section. The most interesting part of this methodology is the automatic modeling of branching when an action depletes the number of objects of a role. Since we abstracted away the true numbers, our model must reflect the possibility that the object removed from a role could be the last one playing that role. Fig.2 shows an example of this situation in the blocks world where the focus operation splits the abstract structure into two relevant cases.

Our methodology for finding plans uses the abstraction mechanism described above to construct an abstract state space. The abstract start state represents a set of concrete states from problem instances with varying numbers of objects. We perform a search in the abstract state space using the action model described. Typically, back edges and loops are encountered. Unlike a search in the concrete state space, not all loops encountered here are stagnant – on the contrary, part of our goal is to identify paths with loops that make progress and lead to the goal. Once such a path is found, we find the preconditions on the concrete states for which it will work, and annotate the start structure to reflect the availabil-

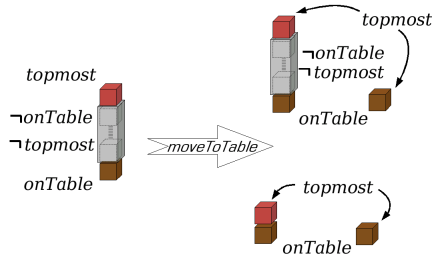


Figure 2: Branching. In blocks world the focus operation models “drawing” an object from a role. In doing so it produces a branch on the number of objects left in the  $\neg\text{topmost} \wedge \neg\text{onTable}$  role.

ity of this partial solution. We then repeat this process until either the entire abstract start structure has been covered, or all the interesting paths have been analyzed. In this paper, we only consider paths with simple, i.e. non-nested loops. Our algorithm for learning plans is a simple modification of this procedure, described in its own section.

## Framework for Canonical Abstraction

We assume that actions are deterministic and that their results are observable. States of a domain are represented by two-valued logical structures consisting of a universe of constant elements or objects, and definitions for all the predicates in a domain-specific vocabulary. We use  $\llbracket \varphi \rrbracket^S$  to denote the truth value of a closed formula  $\varphi$  in the structure  $S$ . State transitions are carried out using an action’s *transformer*, which is a set of first-order formulas describing new values of every predicate in terms of the old values. We use three-valued logic structures, which we call abstract structures, to represent sets of two-valued structures succinctly.

**Example 1** A typical blocks world vocabulary would consist of a binary relation *on*; this can be used to define other relations like *onTable* and *topmost* using first-order formulas. For clarity in presentation however, we will treat all of these relations as separate and equally fundamental. An example structure,  $S$ , in this vocabulary can be described as: the universe,  $|S| = \{b_1, b_2, b_3\}$ ,  $\text{onTable}^S = \{b_3\}$ ,  $\text{topmost}^S = \{b_1\}$ ,  $\text{on}^S = \{(b_1, b_2), (b_2, b_3)\}$ .

We assume actions to be deterministic. The *action transformer* for an action  $a$  (written  $\tau_a$ ) consists of a set of preconditions and a set of formulas defining the new value  $r'$  of each relation  $r$ . We separate these two parts of an action transformer: the argument selection and precondition checks are done in a pre-action step. For instance, for the *move* action, the pre-action steps set up predicates identifying the object to be moved and its destination. These predicates are used to bind the two variables  $\text{obj}_1$  and  $\text{obj}_2$  to the block to be moved, and its destination, respectively. Preconditions of actions may enforce *integrity constraints*, for example, the precondition for *move* could be  $\text{topmost}(\text{obj}_1) \wedge \text{topmost}(\text{obj}_2) \wedge \text{obj}_1 \neq \text{obj}_2$  ensuring that the relation *on* remains 1:1 and irreflexive.

We write  $\tau_a(S)$  to denote the structure obtained by applying action  $a$  to structure  $S$ . Let,  $\tau_a(\Gamma) = \{\tau_a(S) \mid S \in \Gamma\}$  be the application of  $a$  to a set of structures,  $\Gamma$ .

Let  $\Delta_i^+$  ( $\Delta_i^-$ ) be formulas representing the conditions under which the relation  $r_i(\bar{x})$  will be changed to true (false)

by a certain action. The formula for  $r'_i$ , the new value of  $r_i$ , is written in terms of the old values of all the relations:

$$r'_i(\bar{x}) = (\neg r_i(\bar{x}) \wedge \Delta_i^+) \vee (r_i(\bar{x}) \wedge \neg \Delta_i^-) \quad (1)$$

The RHS of this equation consists of two clauses, the first of which holds for arguments on which  $r_i$  is changed to true by the action; the second clause holds for arguments on which  $r_i$  was already true, and remains so after the action.

**Example 2** In the blocks world, action *move* has two arguments:  $\text{obj}_1$ , the block to be moved, and  $\text{obj}_2$ , the block it will be placed on. Update formulas for *on* and *topmost* are:

$$\begin{aligned} \text{on}'(x, y) &= \neg \text{on}(x, y) \wedge (x = \text{obj}_1 \wedge y = \text{obj}_2) \\ &\vee \text{on}(x, y) \wedge (x \neq \text{obj}_1 \vee y = \text{obj}_2) \\ \text{topmost}'(x) &= \neg \text{topmost}(x) \wedge (\text{on}(\text{obj}_1, x) \wedge x \neq \text{obj}_2) \\ &\vee \text{topmost}(x) \wedge (x \neq \text{obj}_2) \end{aligned}$$

The goal condition is represented as a first-order formula; given a start structure, the objective of planning is to reach a structure that satisfies the goal condition. With this notation, we define a domain-schema as follows:

**Definition 1** A *domain-schema* is a tuple  $\mathcal{D} = (\mathcal{V}, \mathcal{A}, \varphi_g)$  where  $\mathcal{V}$  is a vocabulary,  $\mathcal{A}$  a set of action transformers, and  $\varphi_g$  a first-order formula representing the goal condition.

Some special unary predicates are classified as *abstraction predicates*. The special status of these predicates arises from the fact that when we perform abstraction, unlike other predicates, these predicates do not become ambiguous. We define the *role* an element plays as the set of abstraction predicates it satisfies:

**Definition 2** A *role* is a conjunction of literals consisting of every abstraction predicate or its negation.

**Example 3** In the blocks world, with abstraction predicates *topmost* and *onTable*, the role  $\neg\text{topmost} \wedge \neg\text{onTable}$  designates blocks that are in the middle of a stack.

## Canonical Abstraction

Canonical abstraction [Sagiv *et al.*, 2002] groups states together by only counting the number of objects in a role up to two. If a role contains more than one element, they are combined into a *summary element*. We can tune the choice of abstraction predicates so that abstract structures effectively model some interesting general planning problems and yet the size and number of abstract structures remains manageable.

The imprecision that must result when states are merged is modeled using three-value logic. In a three-valued structure the possible truth values are  $0, \frac{1}{2}, 1$ , where  $\frac{1}{2}$  means “don’t know”. If we order these values as  $0 < \frac{1}{2} < 1$ , then conjunction evaluates to minimum, and disjunction evaluates to maximum. See Fig.1 where *on* holds between the topmost block,  $e_1$ , and some but not all of the blocks of the summary element,  $e_2$ . Thus the truth value of  $\text{on}(e_1, e_2)$  is  $\frac{1}{2}$ , drawn in TVLA as a dotted arc.

We next define *embeddings* [Sagiv *et al.*, 2002]. Define the *information order* on the set of truth values as  $0 < \frac{1}{2}, 1 < \frac{1}{2}$ , so lower values are more precise. Intuitively,  $S_1$  is embeddable in  $S_2$  if  $S_2$  is a correct but perhaps less

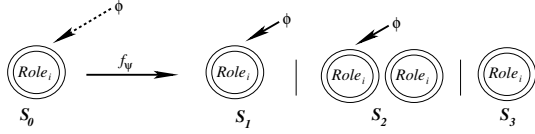


Figure 3: Effect of focus with respect to  $\phi$ .

precise representation of  $S_1$ . In the embedding, several elements of  $S_1$  may be mapped to a single summary element in  $S_2$ .

**Definition 3** Let  $S_1$  and  $S_2$  be two structures and  $f : |S_1| \rightarrow |S_2|$  be a surjective function.  $f$  is an *embedding* from  $S_1$  to  $S_2$  ( $S_1 \sqsubseteq^f S_2$ ) iff for all relation symbols  $p$  of arity  $k$  and elements,  $u_1, \dots, u_k \in |S_1|$ ,  $\llbracket p(u_1, \dots, u_k) \rrbracket^{S_1} \prec \llbracket p(f(u_1), \dots, f(u_k)) \rrbracket^{S_2}$ .

The universe of the canonical abstraction,  $S'$ , of structure,  $S$ , is the set of nonempty roles of  $S$ . In order to merge all elements that have the same role, we use the subscript  $\{p \in A \mid \llbracket p(x) \rrbracket^{S, u/x} = 1\}$ ,  $\{p \in A \mid \llbracket p(x) \rrbracket^{S, u/x} = 0\}$  to denote elements in the abstracted domain.

**Definition 4** The embedding of  $S$  into its *canonical abstraction* wrt the set  $A$  of abstraction predicates is the map:

$$c(u) = e_{\{p \in A \mid \llbracket p(x) \rrbracket^{S, u/x} = 1\}, \{p \in A \mid \llbracket p(x) \rrbracket^{S, u/x} = 0\}}$$

Further, for any relation  $r$ , we have  $\llbracket r(e_1, \dots, e_k) \rrbracket^{S'} = l.u.b_{\prec} \{ \llbracket r(u_1, \dots, u_k) \rrbracket^S \mid c(u_1) = e_1, \dots, c(u_k) = e_k \}$ .

The truth values in canonical abstractions are as precise as possible: if all embedded elements have the same truth value then this truth value is preserved, otherwise we must use  $\frac{1}{2}$ . The set of concrete structures that can be embedded in an abstract structure  $S$  is called the *concretization* of  $S$ :  $\gamma(S) = \{S' \mid \exists f : S' \sqsubseteq^f S\}$ .

**Focus** With such an abstraction, the update formulas for actions might evaluate to  $\frac{1}{2}$ . We therefore need an effective method for applying action transformers while not losing too much precision. This is handled in TVLA using the *focus* operation. The focus operation on a three-valued structure  $S$  with respect to a formula  $\phi$  produces a set of structures which have definite truth values for every possible instantiation of variables in  $\phi$ , while collectively representing the same set of concrete structures,  $\gamma(S)$ . A focus operation with a formula with one free variable is illustrated in Fig.3: if  $\phi()$  evaluates to  $\frac{1}{2}$  on a summary element,  $e$ , then either all of  $e$  satisfies  $\phi$ , or part of it does and part of it doesn't, or none of it does. This process could produce structures that are inherently infeasible. Such structures are either refined or discarded during TVLA's *coerce* operation using a set of restricted first-order formulas called *integrity constraints*. In Fig.3 for instance, if integrity constraints restricted  $\phi$  to be unique and satisfiable, then structure  $S_3$  in Fig.3 would be discarded and the summary elements for which  $\phi()$  holds in  $S_1$  and  $S_2$  would be replaced by singletons.

The focus operation wrt a set of formulas works by successive focusing wrt each formula in turn. The result of the focus operation on  $S$  wrt a set of formulas  $\Phi$  is written  $f_\Phi(S)$ . We use  $\psi_a$  to denote the set of focus formulas for action  $a$ .

## Choosing Action Arguments

Usually, action specifications are allowed to have free variables. During a typical TVLA execution, such an action is tried with every binding of the free variables that satisfies the pre-conditions. In static analysis this feature can be used to model non-determinism. Our algorithm chooses the arguments in a series of pre-action focus steps. For example, to choose  $obj_1$ , in the *move* action we would focus on an auxiliary unary predicate  $obj_1'()$  that is constrained to be single-valued and to imply *topmost*.

## Action Application

Recall that the predicate update formulas for an action transformer take the form shown in equation 1. This equation might evaluate to indefinite truth values in abstract structures. For our purposes, the most important updates are for (unary) abstraction predicates since precision in their values determines the accuracy of modeling action dynamics. In this special case, the expressions for  $\Delta_i^+$  and  $\Delta_i^-$  are *monadic* (i.e. have only one free variable apart from the action arguments which are bound by the pre-action steps).

In order to obtain definite truth values for these updates, we focus the given abstract structure  $S$  using focus formulas  $\Delta_i^\pm$ . Once the action transformer has been applied, we apply canonical abstraction (this is called “blur” in TVLA) on the resulting structures to get the abstract result structures.

## Transitions

Once the action arguments have been chosen, there are three steps involved in action application: action specific focus, action transformation, and blur. The transition relations  $\xrightarrow{a}$ , captures the combined effect of these steps:

**Definition 5 (Transition Relation)**  $S_1 \xrightarrow{a} S_2$  iff  $S_1$  and  $S_2$  are three-valued structures and there exists a focused structure  $S_1^i \in f_{\psi_a}(S_1)$  s.t.  $S_2 = blur(\tau_a(S_1^i))$ .

Sometimes however we will need to study the exact path  $S_1$  took in getting to  $S_2$ . For this, the transition  $S_1 \xrightarrow{a} S_2$  can be decomposed into a set of transition sequences  $\{(S_1 \xrightarrow{f_{\psi_a}} S_1^i \xrightarrow{\tau_a} S_2^i \xrightarrow{b} S_2) \mid S_1^i \in f_{\psi_a}(S_1) \wedge S_2^i = \tau_a(S_1^i) \wedge S_2 = blur(S_2^i)\}$ .

**Transition Graphs** Transition graphs are graphs of all the transition relations  $\xrightarrow{a}$  with nodes representing abstract structures. Formally,

**Definition 6** Given a domain  $\mathcal{D}$  and an initial state  $S_0$ , the *transition graph for  $\mathcal{D}$  starting with  $S_0$* ,  $\mathcal{G}_{\mathcal{D}}(S_0)$  is the edge-labeled multigraph defined by the set of relations  $\{ \xrightarrow{a} : a \in \mathcal{A} \}$  on the set of structures reachable from  $S_0$ . Each edge of  $\mathcal{G}_{\mathcal{D}}(S_0)$  is labeled by the appropriate action.

We omit the subscript  $\mathcal{D}$  where it is understood from context. The entire transition graph of a domain is a finite structure and can be generated using TVLA.

## Finding Pre-conditions

The idea behind our algorithm for generalized planning is to successively find paths in a transition graph and for each such path, to compute the pre-conditions under which it takes concrete structures to structures satisfying the goal.

In order to accomplish this, we need a way of representing subsets of abstract structures that are guaranteed to take a particular branch of an action’s focus operation. Next, we need to propagate these subsets backwards through action edges in the given path all the way up to the start structure.

We represent subsets of an abstract structure by annotating a structure with a set of conditions from a chosen constraint language. In static analysis terms, we use annotations to *refine* our abstraction. Formally,

**Definition 7 (Annotated Structures)** Let  $\mathcal{C}$  be a language for expressing constraints on three-valued structures. A  $\mathcal{C}$ -annotated structure  $S|_{\mathcal{C}}$  is the refinement of  $S$  consisting of structures in  $\gamma(S)$  that satisfy the condition  $C \in \mathcal{C}$ . Formally,  $\gamma(S|_{\mathcal{C}}) = \{s \in \gamma(S) \mid s \models C\}$ .

We extend the notation defined above to sets of structures, so that if  $\Gamma$  is a set of structures then by  $\Gamma|_{\mathcal{C}}$  we mean the structures in  $\Gamma$  that satisfy  $C$ . Thus we have  $\gamma(S|_{\mathcal{C}}) = \gamma(S)|_{\mathcal{C}}$ .

The (annotated) pre-image of an annotated structure gives us the pre-condition for reaching that structure. If finding this pre-image is possible, we say the domain is *amenable to back propagation*:

**Definition 8 (Annotated Domains)** An *annotated domain-schema* is a pair  $\langle \mathcal{D}, \mathcal{C} \rangle$  where  $\mathcal{D}$  is a domain-schema and  $\mathcal{C}$  is a constraint language. An annotated domain-schema is *amenable to back-propagation* if for every transition  $S_1 \xrightarrow{f\psi_a} S_1^i \xrightarrow{\tau_a} S_2^i \xrightarrow{b} S_2$  and  $C_2 \in \mathcal{C}$  there exists  $C_1^i \in \mathcal{C}$  such that  $\tau_a(\gamma(S_1)|_{C_1^i}) = \tau_a(\gamma(S_2^i)|_{C_2})$ .

In terms of this definition, since  $\tau_a(\gamma(S_2^i))$  is the subset of  $\gamma(S_2^i)$  that has pre-images in  $S_1^i$  under  $\tau_a$ ,  $S_1|_{C_1^i}$  is the pre-image of  $S_2|_{C_2}$  under a particular focused branch (the one using  $S_1^i$ ) of action  $a$ . The disjunction of  $C_1^i$  over all branches taking  $S_1$  into  $S_2$  therefore gives us a more general annotation which is not restricted to a particular branch of the action update. Using the abbreviation  $\tau_{k\dots 1}$  to represent the successive application of action transformers  $a_1$  through  $a_k$  in this order, we get:

**Proposition 1 (Linear backup)** Suppose  $\langle \mathcal{D}, \mathcal{C} \rangle$  is an annotated domain-schema that is amenable to back-propagation and  $S_1, \dots, S_k \in \mathcal{D}$  are distinct structures such that  $S_1 \xrightarrow{\tau_1} S_2 \dots \xrightarrow{\tau_{k-1}} S_k$ . Then for all  $C_k$  there exists  $C_1$  such that  $\tau_{k-1\dots 1}(\gamma(S_1)|_{C_1}) = \tau_{k-1\dots 1}(\gamma(S_k)|_{C_k})$ .

The restriction of distinctness in this proposition confines its application to action sequences without loops.

**Inequality-Annotated domain-schemas** Let us denote by  $\#_R(S)$  the number of elements of role  $R$  in structure  $S$ . We use  $\mathcal{C}_I(\mathcal{R})$ , the language of constraints expressed as sets of linear inequalities using  $\#_{R_i}(S)$ , for annotations.

**Quality of Abstraction** In order for us to be able to classify the effects of focus operations, we need to impose some quality-restrictions on the abstraction. Our main requirement is that the changes in abstraction predicates should be characterized by roles: given a structure, an action can change a certain abstraction predicate only for objects with a certain role. We formalize this property as follows: a formula  $\varphi(x)$  is said to be *role-specific* in  $S$  iff only objects of a certain role can satisfy  $\varphi$  in  $S$ .

We therefore want our abstraction to be rich enough to make the action change formulas,  $\Delta_i^{\pm}$ , role-specific in every structure encountered. For example, in the blocks world state shown in Fig.2 the *move* action can only change the *topmost* predicate for a block of the role  $\neg\text{topmost} \wedge \neg\text{onTable}$ , representing blocks in the middle of the stack. The design of a problem representation and in particular, the choice of abstraction predicates therefore needs to carefully balance the competing needs of tractability in the transition graph and the precision required for back propagation.

The following definition and theorem identify a class of domains where back propagation is possible.

**Definition 9 (Extended-LL domains)** An *Extended-LL domain* with start structure  $S_{start}$  is a domain-schema such that  $\Delta_i^+$  and  $\Delta_i^-$  are role-specific, exclusive when not equivalent, and uniquely satisfiable in every structure reachable from  $S_{start}$ . More formally, if  $S_{start} \xrightarrow{*} S$  then  $\forall i, j, \forall e, e' \in \{+, -\}$  we have  $\Delta_i^e$  role-specific and either  $\Delta_i^e \equiv \Delta_j^{e'}$  or  $\Delta_i^e \implies \neg\Delta_j^{e'}$  in  $S$ .

**Theorem 1 Extended-LL Domains are amenable to back-propagation.**

Methods described in Srivastava *et al.* [2007] can be used to find plan pre-conditions in extended-LL domains.

Intuitively, these domain-schemas are those where:

1. The information captured by roles is sufficient to determine whether or not an object of any role will change roles due to an action; and
2. The number of objects being acquired or relinquished by any role is fixed (constant) for each action.

Examples of such domains are linked lists, blocks-world scenarios (the appropriate start structures are defined in the section on Examples), problems in the rocket domain, assembly domains where different objects can be constructed from constituent objects of different roles etc.

**Handling Paths with Loops** In extended-LL domains we can also effectively propagate annotations back through paths consisting of simple (non-nested) loops.

**Proposition 2 (Back-propagation through loops)** Suppose  $S_0 \xrightarrow{\tau_1} S_1 \xrightarrow{\tau_2} \dots \xrightarrow{\tau_{n-1}} S_{n-1} \xrightarrow{\tau_0} S_0$  is a loop in an extended-LL domain with a start structure  $S_{start}$ . Let the structures before entering the loop and after exit be  $S$  and  $S_f$ . We can then compute an annotation  $C(l)$  on  $S$  which selects the structures that will be in  $S_f|_{C_f}$  after  $l$  iterations of the loop on  $S$ , plus the simple path from  $S$  to  $S_f$ .

## Algorithm for Generalizing From Examples

In this section we present our approach for computing a generalized plan from a plan that works for a single problem instance. The idea behind this technique is that if a given concrete plan contains sufficient unrollings of some simple loops, then we can automatically identify these loops by observing the action-abstract structure trace of the plan starting with a suitable abstract structure. We can then enhance this plan using the identified loops and use the techniques discussed above to find the set of problem instances for which this new generalized plan will work. The procedure is shown in Algorithm 1. It is described with an example in the next

**Input:**  $\pi = (a_1, \dots, a_n)$ : plan for  $S_0^\#$ ;  $S_i^\# = a_i(S_{i-1}^\#)$

**Output:** Generalized plan  $\Pi$

- 1  $S_0 \leftarrow c(S^\#)$ ;  $\Pi \leftarrow \pi$ ;  $C_\Pi \leftarrow \top$
- 2 Apply transformers for  $\pi$  on  $S_0$  to get  $S'_{i-1}$ ,  $S_i$  s.t.  
 $S'_{i-1} \in f_{a_i}(S_{i-1})$ ,  $\tau_i(S'_{i-1}) = S_i$  and  $S_i^\# \sqsubseteq S_i$ .
- 3 **if**  $\exists C \in C_I(\mathcal{R}) : S_n|_C \models \varphi_g$  **then**
- 4      $\Pi \leftarrow \text{formLoops}(S_0 : a_1 \dots, S_{n-1} : a_n, S_n)$
- 5      $C_\Pi \leftarrow \text{findPrecon}(S_0, \Pi, \varphi_g)$
- 6 **end**
- 7 **return**  $\Pi, C_\Pi$

**Algorithm 1:** GeneralizeExample

section; we describe the main subroutines here. Suppose we are given a concrete example plan  $\pi$  for a structure  $S_0^\#$ . We work with  $S_0$ , an abstract structure including  $S_0^\#$ .  $S_0$  can be any structure which makes the resulting domain extended-LL, and for which we need a generalized plan; the canonical abstraction of  $S_0^\#$  forms a natural choice.

The *formLoops* subroutine converts a linear path of structures and actions into a path with simple loops. One way of implementing this routine is by making a single pass over the input path, and adding back edges whenever a structure  $S_j$  is found such that  $S_i = S_j (i < j)$ , and  $S_i$  is not part of, or behind a loop. Structures and actions following  $S_j$  are merged with those following  $S_i$  if they are identical; otherwise, the loop is exited via the last action edge. This method produces one of the possibly many simple-loop paths from  $\pi$ ; we could also produce all such paths. *formLoops* thus gives us a generalization  $\Pi$  of  $\pi$ . We then use the *findPrecons* subroutine to obtain the restriction on  $S_0$  for which  $\Pi$  works.

**Example**

We illustrate this idea using an example in the blocks world domain. Given a stack consisting of red blocks below and blue blocks above, we need to find a plan for constructing a stack with alternating red and blue blocks above an assigned red, *base* block with a blue block on top. The number of red and blue blocks in the given stack is unknown.

**Representation** Our vocabulary consists of the predicates  $\{t[on]^2, on^2, topmost^1, onTable^1, obj_1^1, obj_2^1, red^1, blue^1, base^1, misplaced^1\}$ , where all the unary predicates are abstraction predicates. The *misplaced* predicate is used to check that a block is above a stack of alternating colors.  $t[on]$  is the transitive closure of *on*, and is used in integrity constraints for the coerce operation. The  $obj_1, obj_2$  predicates are used to select action arguments before an action is applied. There are two actions: *move()*, which places  $obj_1$  on top of  $obj_2$  and *moveToTable()*, which moves  $obj_1$  to the table. For simplicity of demonstration, we restrict to only one stack on the table in this example (for discussion on multiple stacks, see Srivastava *et al.* [2007]). This representation can also be used to describe linked lists and subsequently for creating or learning algorithms for some programming tasks.

Fig.5 shows  $\Pi_G$ , a generalized plan for achieving the desired configuration. This plan is the goal for our learning algorithm, and we show how to learn it from a concrete plan for a fixed instance of the problem. In fact our techniques can generate  $\Pi_G$  itself from scratch - we briefly discuss this

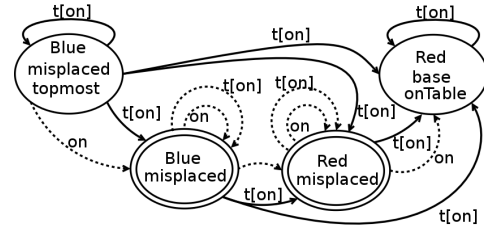


Figure 4: Initial structure for striped blocks world.

- |                                      |                               |
|--------------------------------------|-------------------------------|
| 1 Move mispld. blk to table          | //All blocks on table         |
| 2 while #(blue blocks in middle) > 1 | 9 Move blue blk to stack      |
| Move mispld. blk to table            | 10 Move red blk to stack      |
| 3 Move mispld. blk to table          | 11 Move blue blk to stack     |
| 4 Move mispld. blk to table          | 12 Move red blk to stack      |
| //All Blue blocks on table           | 13 Move blue blk to stack     |
| 5 Move mispld. blk to table          | 14 while #(blue on table > 1) |
| 6 while #(red blocks in middle) > 1  | Move red blk to stack         |
| Move mispld. blk to table            | Move blue blk to stack        |
| 7 Move mispld. blk to table          | 15 Move red blk to stack      |
| 8 Move mispld. blk to table          | 16 Move blue blk to stack     |

Figure 5: A generalized plan with three loops.

in the next section.

Suppose we are given a concrete plan  $\pi = a_1, \dots, a_n$  for the initial structure  $S_0^\#$  consisting of a stack with 5 blue blocks above 5 red blocks.  $\pi$  consists of the following operations: 5 actions moving blue blocks to the table, followed by 4 actions moving red blocks to the table, and finally by 9 actions moving red and blue blocks alternately back to the stack. This plan can be computed by any classical planner. Fig. 4 shows  $S_0$ , the canonical abstraction for  $S_0^\#$ .  $\gamma(S_0)$  contains infinitely many stacks with any number of blue blocks upon any number of red blocks.

Let  $S_i^\# = a_i(S_{i-1}^\#), i > 0$ . To obtain the general plan, we first convert  $\pi$  into a sequence of action transformers by replacing each action with a transformer that uses as its argument(s) any block having the role of the real action's argument(s). We then apply the resulting sequence of transformers on  $S_0$ . This gives us a sequence of sets of structures representing the possibilities after each action. From each set in the sequence, we select the structure  $S_i$  such that  $S_i^\# \sqsubseteq S_i$ . Let us call the sequence  $\{(S_0, a_0), \dots, (S_i, a_i), (S_{n-1}, a_{n-1})\} \pi_{abs}$ , where  $S_n$  is the final structure.

$\Pi_{abs}$  is an unrolled version  $\Pi_G$  as applied on  $S_0$ . Further, every unrolled loop in  $\Pi_{abs}$  is punctuated by a repeated abstract structure. Therefore, when *formLoops* is executed on  $\Pi_{abs}$ , it finds these repeated structures and re-creates the loops, giving us exactly  $\Pi_G$ ! As can be observed by the structure of  $\Pi_G$  itself,  $S_n$  satisfies the goal and we use the techniques presented in this paper to find the annotation on  $S_0$  which  $\Pi_G$  will work.

Our algorithm computes the desired annotation by associating counters  $B_t(R_t)$  with roles corresponding to *blue(red)*, but not *base*) blocks that are *onTable* and *topmost* and  $B_m(R_m)$  with *blue(red)* blocks that are neither *topmost* nor *onTable*. The obtained annotation at structure  $S_0$  is



$$[B_t = l - l_b, R_t = l - l_r - 1, B_m = 3 + l_b, R_m = 4 + l_r],$$

where the  $l_r(l_b)$  denote number of iterations of the unstacked (unstack-blue) loop. Together these conditions imply that the total number of blue blocks (counting the topmost blue block and bottommost red block) is  $4 + l$ , exactly equal to the number of red blocks in the initial structure.

We are thus able to extract the generalized plan  $\Pi_G$  from  $\pi$ , a simple concrete plan that could have been found by any classical planner. We also find the pre-conditions, or problem instances for which our learned plan achieves the goal.

### Algorithm for Finding Generalized Plans

Let the annotated-domain-schema  $\langle \mathcal{D}, C_I \rangle$  be an extended-LL domain with a start structure  $S_0$ . We can then formulate an algorithm for generalized planning as follows. We proceed in phases of search and annotation on the transition graph. During the search phase we find a path  $\pi$  from the start structure to structures satisfying the goal condition. The transition graph could also be dynamically generated during this phase. During the annotation phase, we find the annotation on  $S_0, C_\pi$  for  $\pi$ . If  $C_\pi$  adds to the structures in  $S_0$ , we include it in the general plan, and continue the search. The resulting algorithm can be implemented in an any-time fashion, by outputting plans capturing more and more problem instances as new paths are found in the transition graph.

Propositions 2 and 1 thus give us the following theorem:

**Theorem 2** (Generalized planning for extended-LL domains) *For an extended-LL domain with a start structure  $S_0$  it is possible to find plans  $\pi_i$  and annotations  $C_i$  such that  $\forall s \in \gamma(S|_{C_i}), \pi_i$  takes  $s$  to the goal; further, for  $s \in \gamma(S) \setminus \gamma(S|_{\bigcup_i C_i}),$  the goal is not reachable via plans with only simple loops in the transition graph  $\mathcal{G}(S_0)$ .*

### Example: Striped Blocks World

Our goal is to find a generalized plan for the abstract problem of constructing a stack of blocks of alternating colors as described in the previous section.

**Input** An abstract structure (Fig.4) representing all problem instances with a single stack of unknown and unbounded numbers of red and blue blocks (at least two of each), with the red blocks below the blue blocks (Fig. 4). The goal condition is  $\forall x \text{-misplaced}(x)$ .

**Output** Our algorithm finds a generalized plan for solving *infinitely many* instances of the given abstract problem. In fact, in this case we get a solution for all solvable instances.  $\Pi_G$  (Fig.5) shows the steps from the most interesting branch of the computed plan. The pre-condition annotation for  $\Pi_G$  was discussed in the previous section. Cases with fewer number of blocks are captured by other paths (found before this path), and their pre-conditions are obtained similarly.

### Related Work

Interest in computing plan generalizations in AI is perhaps as old as planning itself. Attempts at producing plans for more than a single problem instance started with Fikes *et al.* [1972]. Their framework parametrized and indexed subsequences of existing plans for use as macro operations or alternatives to failed actions. However, this approach turned out to be quite limited and prone to over-generalization.

This initial effort was followed by various approaches to plan reuse: *case based planning* [Hammond, 1996], *analogical reasoning* [Veloso, 1994] and more recently, domain-specific planning templates [Winner & Veloso, 2003]. Winner & Veloso [2003] provide an approach for parametrizing, extracting and assimilating example plans into templates. However this technique does not guarantee correctness and is limited in its capacity to detect loops. Levesque [2005] presents an interesting method for creating general plans by iteratively generating plans and finding repetitive patterns involving a unique *planning parameter*. Our roles are more generalized forms of such parameters. Also, unlike his approach, we find provably correct plans. However, our approach cannot currently handle numeric fluents, which are accommodated in Levesque's approach.

### Conclusion and Future Work

We present a new unified framework for generalized planning using abstraction across problem instances for learning and even constructing generalized plans. The main contributions of this paper are the presentation of an abstraction technique particularly conducive for this purpose, the formulation of planning and learning algorithms, and a precise analytical characterization of a setting in which they are guaranteed to work. Our planning algorithm is only partially implemented; a full implementation requires interfacing with TVLA and is left for future work. We use several examples to illustrate how our algorithms work and to demonstrate the overall power of this approach.

Our framework opens up several interesting research avenues for future work. Searching in an abstract space that spans different problem instances presents new challenges for heuristic and pruning techniques. We are exploring natural extensions such as generalizing our techniques to a wider class of domains and to plans with nested loops. Finally, other annotation languages could be examined as they may provide a way of moving beyond extended-LL domains.

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